

# A model of karst systems of the Usturt plateau (Middle Asia)

A.S. Victorov

The Institute of Environmental Geosciences RAS, P.O. Box 145, 13, Ulansky per., 101 000, Moscow, Russia.  
dist@geoenv.ru; vic\_as@mail.ru

## ABSTRACT

The issues relating to the structure and development of desert karst is currently one of the least studied problems. The goal of the research efforts presented in this paper was to create a model of karst systems in the Usturt Plateau and, primarily, of the systems formed by the prevailing numerous surface karst landforms. The model is based on the following main assumptions: the process of emergence of the karst phenomena is probabilistic and occurs independently at non-intersecting sites; the probability of emergence of one depression at a reference site depends only on its area. The increase of the dimensions of a karst landform is a random process that occurs independently of other karst landforms, its speed is governed by random factors, and it is directly proportionate to the existing dimensions of the landform. The model does not assume constancy of climatic characteristics. The testing was conducted at several reference sites of the Usturt Plateau. The research allowed us to reach the following conclusions: the karst depressions of the Usturt Plateau, which are the typical forms of desert karst, are governed by the Poisson distribution of their centres and by the lognormal distribution of their dimensions, similar to the karst depressions in a humid climate. The results may be used in assessments of karst hazard for linear and areal structures, as well as for small-dimensional structures.

Key words: stochastic processes, karst and suffusion depressions, uniform plateau site

## ***Un modelo de sistemas kársticos de la meseta Usturt (Ásia central)***

### RESUMEN

*Las cuestiones relacionadas con la estructura y el desarrollo del karst desértico son en la actualidad uno de los problemas menos estudiados. El objetivo de los esfuerzos de investigación presentados en este documento ha sido el crear un modelo de los sistemas kársticos Usturt Plateau y principalmente de los sistemas formados por las numerosas formas kársticas superficiales. El modelo se basa en las siguientes hipótesis principales: el proceso de la aparición de fenómenos kársticos es probabilístico y se produce de forma independiente en los sitios que no se intersectan; y la probabilidad de aparición de una depresión en un sitio de referencia depende sólo de su área. El aumento de las dimensiones de una forma kárstica es un proceso aleatorio que se produce independientemente de otras formas kársticas, su velocidad se rige por factores aleatorios, y es directamente proporcional a las dimensiones existentes de la forma kárstica. El modelo no asume la constancia de las características climáticas. La prueba se llevó a cabo en varios sitios de referencia de la meseta de Usturt. La investigación realizada permite llegar a las siguientes conclusiones: las depresiones kársticas de la meseta de Usturt, que son las formas típicas del karst desértico, se rigen por la distribución de Poisson de sus centros y por la distribución lognormal de sus dimensiones, similar a las depresiones kársticas de clima húmedo. Los resultados obtenidos pueden ser utilizados en las evaluaciones del riesgo kárstico en estructuras lineales y areales, así como para estructuras de pequeñas dimensiones.*

*Palabras clave: procesos estocásticos, karst y depresiones de sufusión, meseta uniforme.*

## VERSIÓN ABREVIADA EN CASTELLANO

### Introducción

Las cuestiones relacionadas con la estructura y el desarrollo del karst desértico son en la actualidad unos problemas poco estudiados en la investigación del karst. Una de las regiones más escasamente estudiadas donde se desarrolla el karst desértico es la Meseta de Usturt. Usturt representa una vasta formación de tierras altas de tipo meseta, entre el Aral y el Mar Caspio. El relieve Usturt es básicamente plano o tiene suaves colinas compuestas de roca caliza con depósitos de margas sarmatienses encajadas en amplias depresiones internas (Shordzha, Shakhpakhty, etc.). Los relieves kársticos en Usturt están representados por los siguientes tipos: depresiones takyr pequeñas, dolinas kársticas («ahn»), y valles kársticos. Los procesos kársticos son los responsables de la formación de las depresiones internas. Creemos que numerosas depresiones ocupadas por takyrs son relieves kársticos más desarrollados. Este punto de vista también se ha adoptado por un importante número de investigadores (BA Fedorovich, EA Agbalyants, SV Victorov, etc.) Se basa en el hecho de que los sedimentos de carbonato cerraron depresiones en la superficie enrasada de la Meseta, ya que al parecer, no pueden ser creados por cualquier otro proceso de denudación (erosión, procesos eólicos, etc.). Los esfuerzos de investigación más significativos en el karst de la Meseta de Usturt están utilizando sólo los métodos tradicionales y no intentan crear un modelo matemático del desarrollo del sistema kárstico. El objetivo de los esfuerzos de investigación presentados en este trabajo es crear un modelo de sistemas kársticos, y principalmente de los sistemas formados por las numerosas geoformas kársticas dominantes en superficie. El modelo utiliza los planteamientos de la teoría de los procesos estocásticos. También utiliza los resultados de la investigación sobre las leyes que rigen las dimensiones y la distribución espacial de los relieves kársticos exógenos en diferentes territorios (región de los Urales, llanura europea oriental, llanura Mitchell en Indiana, América del Norte, etc.). La mayor parte de los territorios estudiados se encuentran en zonas de clima húmedo. Las regiones áridas están muy poco estudiadas.

Consideremos el proceso de desarrollo de las formas del relieve kárstico en una meseta uniforme. El modelo se basa en los siguientes supuestos principales:

1. El proceso de la aparición de los fenómenos kársticos es probabilístico y se produce de forma independiente en los sitios que no se intersectan; la probabilidad de aparición de una depresión en un sitio de referencia depende sólo de su área ( $\Delta s$ ); y para los sitios pequeños es mucho mayor que la probabilidad de la aparición de varias depresiones.

2. El aumento de las dimensiones de una forma del relieve kárstico es un proceso aleatorio que se produce independientemente de otros relieves kársticos; su velocidad se rige por factores aleatorios y es directamente proporcional a las dimensiones existentes de la forma de relieve que definen la concentración del flujo de agua en ella.

La primera hipótesis parece natural, ya que se deriva de la suposición de la homogeneidad del territorio en cuestión y refleja una escasez relativa de la aparición de depresiones kársticas, es decir, en cualquier sitio sólo hay un número finito de depresiones kársticas (más concretamente de sus centros). El segundo supuesto también parece justificado. La disolución de los sedimentos que se traduce en la aparición de depresiones kársticas en condiciones uniformes en diferentes lugares se produce en paralelo, pero de forma independiente. Además la dependencia directa de la concentración de descarga que define la velocidad del crecimiento del límite en las dimensiones ya establecidas de la depresión es bastante natural. Es importante señalar que el modelo no asume la constancia de las características climáticas.

Los supuestos del modelo permiten obtener analíticamente los patrones de la estructura de las llanuras kársticas. Se puede demostrar que a partir de la validez de estas consideraciones, se consigue estrictamente que la distribución del número de depresiones kársticas (centros) en un sitio elegido al azar se rige por la ley de Poisson, es decir:

$$P(k) = \frac{(\gamma s)^k}{k!} e^{-\gamma s}$$

donde  $\gamma$  es el número medio de depresiones por unidad de área and  $s$  es el área de un sitio de referencia.

El radio de crecimiento de las formas kársticas se puede analizar utilizando las asunciones del modelo y teniendo en cuenta los cambios climáticos. El análisis matemático, y utilizando el teorema del límite central para el crecimiento del radio de las formas kársticas, muestra que incluso en condiciones climáticas cambiantes, la distribución del radio, y por consiguiente, de las áreas de las formas kársticas está gobernada por la ley lognormal, sin embargo con un crecimiento, no lineal sino lineal a trozos, de los parámetros. Si por simplicidad, se asume que las depresiones kársticas primarias en el instante de su emergencia tienen un diámetro de uno, entonces para cualquier momento se debería observar una distribución lognormal de los diámetros de las formas kársticas,

$$F_d(x, t) = \Phi\left(\frac{\ln x - at}{\sigma\sqrt{t}}\right)$$

Donde  $F_d(x, t)$  es la distribución de diámetros,  $\Phi(t)$  es la función de Laplace (la función de distribución acumulada gaussiana), y  $a$  y  $\sigma$  son los parámetros de la distribución, y  $t$  es el tiempo transcurrido después del inicio del proceso.

### Método

La prueba del desarrollado del modelo se llevó a cabo en varios sitios de referencia de la Meseta Usturt. Los sitios 1-4 y 6-8 se encuentran en la planicie inclinada hacia el sur del canto de Karabaur; los sitios 5 y 9 se encuentran en la parte occidental de Usturt, 70 km al sur-oeste de la cordillera. La metodología de prueba incluyó los siguientes pasos:

- Aislamiento de depresiones kársticas,
- Identificación de sus diámetros, diámetros medios, y la ubicación de los centros,
- Verificación de la conformidad de la distribución espacial de los centros de depresión kársticas con la ley de Poisson utilizando el criterio Pearson,
- Verificación de la conformidad de la distribución de los diámetros y diámetros medios de la distribución logarítmica normal utilizando el criterio Pirson.

El aislamiento de las depresiones kársticas en los sitios 5 y 9 se llevó a cabo con la ayuda de imágenes de satélite con una resolución de aproximadamente 2 m. Para los sitios 1-3, además de las imágenes, se utilizó un mapa de los fenómenos kársticos-sufusión, compilado antes (en 1970) por EA Agbalyants y VA Vazerskaya; el mapa se realizó a partir de fotografías aéreas.

El análisis de la verificación empírica de la distribución espacial de las depresiones kársticas en los diferentes sitios, indica que hay un nivel de significación de 0.99 en la hipótesis de la distribución de Poisson de los relieves kársticos en un sitio seleccionado al azar, lo que se corresponde bien con los datos empíricos en tres de cada cuatro sitios. El análisis de los resultados indica también que con un nivel de 0.99 de confianza, la hipótesis sobre la distribución lognormal se corresponde bien con los datos empíricos en tres sitios y sobre la distribución lognormal de los diámetros medios - en otros cuatro sitios.

### Conclusiones

La investigación realizada permite llegar a las siguientes conclusiones:

- Las depresiones kársticas de la meseta Usturt, que son las formas típicas de karst desierto, se rigen por la distribución de Poisson de sus centros y por la distribución logarítmica normal de sus dimensiones; de modo similar a como ocurre con las depresiones kársticas del clima húmedo. Algunas desviaciones pueden ser causados por la falta de homogeneidad de los sitios, por ejemplo, debido a los desplazamientos de falla.
- Las condiciones del desarrollo del Karst en un sitio específico modifican sólo los valores de los parámetros de distribución, mientras que el tipo de la distribución de los centros y dimensiones de las depresiones kársticas permanecen constantes.
- En las condiciones de karst desierto, el modelo propuesto puede ser utilizado para la evaluación de la probabilidad de daño a las estructuras civiles lineales.

### Introduction

The issues relating to the structure and development of desert karst is currently one of the least studied problems. One of the most poorly studied regions where desert karst develops is the Usturt Plateau. Usturt represents a vast plateau-like highland formation between the Aral and the Caspian Seas. The Usturt relief is basically flat or has gentle, rolling hills composed of limestone-marl Sarmatian deposits

incised by wide internal depressions (Sarykamysh, Shordzha, Shakhpakhty, etc.) (Fig.1). At some sites, the limestone alluvium is strongly gypsified. The vegetation consists primarily of *Anabasis salsa*, *Anabasis brachiata*, *Anabasis truncata*, *Artemisia terrae albae*, and sparse brushland (*Haloxylon sarmaticum*). Numerous *takyrs*, i.e., closed depressions with a bare clayey surface, are the characteristic feature of the terrain. The karst landforms on the Usturt are represented by the following types: small *takyr*



**Figure 1:** The Usturt Plateau landscape (photo by M.A. Baldin).  
**Figura 1.** Paisaje de la meseta de Usturt (foto de MA Baldin).



**Figure 2:** A typical takyr (photo by M.A. Baldin).  
**Figura 2.** Un takyr típico (foto de M.A. Baldin).

depressions, doline karst (“ahn”), and karst valleys. Karst processes are the cause of the formation of the internal depressions.

Karst development on the Usturt Plateau has been studied by many scientists. Most research has targeted the diversity and distribution of the karst landforms (Kleiner, 1962, Chikishev, 1971; Goltz, Epifanov, 1965; Kuznetsov, 1965). Many publications deal with the evolution of the vegetation cover in connection with karst development (Momotov, 1953; Victorov, 1971). Thus, the evolution of vegetation cover, namely, the replacement of the *Anabasis salsa* dominant communities at many sites with *Artemisia terrae albae*, has been specifically associated with the development of small karst depressions followed by the demineralization of their soils due to the concentration of surface water discharge into the depressions.

We believe that numerous depressions occupied by *takyrs* are the most widespread karst landforms. This view has also been held by a number of researchers (B.A. Fedorovich, E.A. Agbalyants, S.V. Victorov, etc.) It is based on the fact that closed depressions in carbonate sediments on the leveled surface of the plateau, apparently cannot be created by any other known denudation process (erosion, landslides, aeolian processes, etc.). The surface of the *takyrs* (Fig. 2) is formed in the already existing depressions as the result of the discharge into and accumulation in them of clayey particles and salt. Due to the *takyrs*, the depressions are well identified on remote sensing materials (Fig. 3). In some locations at the surface of the *takyrs*, there are *ponors* – evidence of still occurring karst processes.

*Ahns* are over 15 m deep karst sinkholes (Fig. 4). They often have an asymmetrical and elongated shape; at the bottom of the *ahns*, lakes or *solonchak* often develop (Fig. 5). The asymmetric *ahn* structures and their orientation are associated with their formation on faults and fracture systems. This is clearly seen on satellite images, for example, in the area of the Alan fortress (Eastern Usturt) (Fig. 6).



**Figure 3:** A typical karst depression with takyr development observed on satellite images.  
**Figura 3.** Una depresión kárstica típica con desarrollo de takyr observada desde imágenes de satélite.





**Figure 4:** A karst sinkhole (*ahn*) - a view from an airplane.  
**Figura 4.** Una dolina kárstica («ahn») - vista desde un avión.



**Figure 5:** A karst sinkhole (*ahn*) near the Alan fortress (Eastern Usturt) (photo by V.N. Krasnikov).  
**Figura 5.** Una dolina kárstica («ahn») cerca de la fortaleza de Alan (Este de Usturt) (foto de VN Krasnikov).

The significant research efforts on the Usturt Plateau karst have been using only traditional methods and have not attempted to create a mathematical model of karst system development. The goal of the research efforts presented in this paper was to create a model of karst systems and, primarily, of the systems formed by the prevailing numerous surface karst landforms.

## Methods

The model uses the approaches of the theory of stochastic processes. It also utilizes the results of the research on the laws that govern dimensions and spatial distribution of surface karst landforms in different territories (Ural region, East European Plain, Mitchell Plain [Indiana, North America], etc.) (Tolmachev, 1971; 1990; Victorov, 1979; 1998; 2006; McConnell, Horn, 1972). Most of the studied territories are located in humid climates. Arid regions have been extremely poorly studied.

Let us consider the process of emergence and development of karst landforms on a uniform plateau site. By "uniform" we assume the combination of the following properties: a uniform type of surface relief manifested in uniform patterns of satellite images; constant matter composition of surface sediments and underlying rock; constant, within an entire site, the thickness of surface sediments; the absence of buried hollows; the absence of tectonic dislocations and the location within the same tectonic structure.

The model is based on the following main assumptions:

1. The process of emergence of karst phenomena is probabilistic and occurs independently at non-intersecting sites; the probability of emergence of



**Figure 6:** Karst sinkholes on satellite images of the area near the Alan fortress (Eastern Usturt); straight lines of fractures are clearly visible in the system.  
**Figura 6.** Dolinas kársticas observadas con imágenes de satélite en la zona cercana a la fortaleza Alan (Usturt del Este); las líneas rectas de fracturas son claramente visibles en el sistema.

one depression at a reference site depends only on its area ( $\Delta s$ ) and, for small sites, it is much higher than the probability of the emergence of several depressions; that is:

$$p_1 = \gamma \Delta s + o(\Delta s) \quad (1)$$

$$p_k = o(\Delta s) \quad k = 2, 3, \dots \quad (2)$$

where  $\gamma$  is the average number of depressions per unit area.

2. The increase of the dimensions of a karst landform is a random process that occurs independently of other karst landforms, its speed is governed by random factors, and it is directly proportionate to the existing dimensions of the landform, which define the concentration of water flow in it.

The first assumption seems natural because it follows on from the assumption on the homogeneity of the territory under consideration and reflects a relative sparseness of the emergence of karst depressions, i.e., at any site, there is only a finite number of karst depressions (more precisely, of their centres).

The second assumption also seems justified. Dissolution of sediments that results in the emergence of karst depressions in uniform conditions in different locations occurs in parallel but independently. Besides, the direct dependence of the discharge concentration that defines the speed of the boundary growth on the already established dimensions of the depression is quite natural. This characteristic feature of the discharge concentration largely depends on the climatic and geologic-geomorphological conditions of the territory, though the speed of karst denudation does not depend on these factors.

It is important that the model does not assume constancy of climatic characteristics. The model assumptions allow us to obtain analytically the patterns of the structure of the karst plains. It can be demonstrated (for example, Karlin, 1971) that from the validity of these considerations, it strictly follows that the distribution of the number of karst depressions (centres) at a randomly chosen site is governed by the Poisson law, i.e.,

$$P(k) = \frac{(\gamma s)^k}{k!} e^{-\gamma s} \quad (3)$$

where  $\gamma$  is the average number of depressions per unit area and  $s$  is the area of a reference site.

Let us examine the changes of the dimensions (diameter) of karst depressions. Let us break time interval  $t$  in  $n$  intervals, with the duration of  $t_i, i = 1, \dots, n$ ,

during which the climatic conditions may be assumed as practically homogeneous, and consider a  $i^{th}$  interval ( $\sum_{k=1}^{i-1} t_k, \sum_{k=1}^i t_k$ ). The radius of this karst depression by

the beginning of the analyzed time-interval is  $\xi_{i-1}$ . Let us analyze the development of one depression of the karst origin during  $i^{th}$  time-interval. The second assumption of the model on the proportionality of a random change of the diameter of depressions per  $j^{th}$  year, during this considered time-interval, to the already established diameter, is as follows:

$$\Delta u_j = \xi_j u_j, \quad \sum_{k=1}^{i-1} t_k \leq j \leq \sum_{k=1}^i t_k \quad (4)$$

The factors of proportionality  $\xi_j$  represent random values that depend on atmospheric processes of each specific year, i.e., volume of precipitation, volume of storm run-off, volume of snowmelt, soil temperature, etc. These factors are independent for different years, but are uniformly distributed, respectively,

$$M\xi_j = a_i, \quad D\xi_j = \sigma_i^2. \quad (5)$$

By moving all the variable terms to the left side of equation (4), summing, and replacing the left sum by the integral, we obtain:

$$\int_{x(t_1)}^{x(t_2)} \frac{du}{u} = \sum_{j=t_1}^{t_2} \xi_j, \quad \sum_{k=1}^{i-1} t_k \leq t_1 < t_2 \leq \sum_{k=1}^i t_k \quad (6)$$

where  $x(t_2)$  is the dimension of a depression at moment of time  $t_2$ ,  $x(t_1)$  is the dimension of the depression a moment of time  $t_1$  during the considered time-interval.

Upon integration, we get:

$$\ln x(t_2) - \ln x(t_1) = \sum_{t=t_1}^{t_2} \xi_i. \quad (7)$$

Because according to the central limit theorem (Koroljuk, Portenko, Skorokhod et al, 1985), the sum of a large number of independent components has the normal distribution, the process of growth of the radii of karst landforms at significant  $(t_2-t_1)$  may be approximated as a continuous-time Markov random process with the transition function

$$f(v, x, t) = \frac{1}{\sqrt{2\pi\sigma_i x \sqrt{t}}} e^{-\frac{(\ln \frac{x}{v} - a_i t)^2}{2\sigma_i^2 t}} \quad (8)$$

where  $a_i, \sigma_i$  are the distribution parameters during the considered time-interval,  $v$  is the initial dimension (diameter) of a karst depression,  $x$  is the dimension of the karst depression in time  $t$  (in the considered  $i^{\text{th}}$  time-interval).

In other words, the following expression is true:

$$\zeta(t_i) = \zeta(t_{i-1})\eta_i, \quad (9)$$

where  $\eta_i$  is a random value equal to the ratio between the radii of the karst form at the end of  $\zeta(t_i)$  and the beginning of  $\zeta(t_{i-1})$  for the considered ( $i^{\text{th}}$ ) time-interval. This value, as follows from the expression presented above, has the lognormal distribution with the parameters:

$$M[\ln \eta_i] = a_i t_i, \quad D[\ln \eta_i] = \sigma_i^2 t_i, \quad i = 1, \dots, n-1. \quad (10)$$

From this and assuming that the initial size of the karst landform equals to one, we can get the relation

$$\ln \zeta(t) = \ln \eta_1 + \ln \eta_2 + \dots + \ln \eta_{n-1} + \ln \eta_n, \quad \sum_{i=1}^{n-1} t_i \leq t \leq \sum_{i=1}^n t_i, \quad (11)$$

where  $\zeta(t)$  is the size of the karst landform at moment  $t$ ,  $\eta_i (i=1, \dots, n-1)$  are the log-normally distributed independent random variables with the parameters identified above (10), and  $\eta_n$  is a log-normally distributed independent random variable with the parameters

$$M[\ln \eta_n] = a_n (t - \sum_{i=1}^{n-1} t_i), \quad D[\ln \eta_n] = \sigma_n^2 (t - \sum_{i=1}^{n-1} t_i) \quad (12)$$

From this and considering the sum of independent normally distributed variables, it follows, that the logarithm of the size of the karst landform at the end of time-interval  $(0, t)$  represents a normally distributed random variable with the mathematical expectation

$$M[\ln \zeta(t)] = \sum_{k=1}^{n-1} a_k t_k + a_n (t - \sum_{k=1}^{n-1} t_k) \quad (13)$$

and dispersion

$$D[\ln \zeta(t)] = \sum_{k=1}^{n-1} \sigma_k^2 t_k + \sigma_n^2 (t - \sum_{k=1}^{n-1} t_k) \quad (14)$$

Thus, even in changing climatic conditions, the distribution of the radii and, therefore, of the areas of the karst landforms is governed by the lognormal law, however, not with a linear, but with a piecewise growth of the parameters

If for simplicity, we assume that the primary karst depressions at the moment of their emergence have the diameter of one, then, at any moment of time, we

should observe the lognormal distribution of the diameters of the karst landforms, i.e.,

$$F_d(x, t) = \Phi\left(\frac{\ln x - at}{\sigma\sqrt{t}}\right) \quad (15)$$

or, for the distribution density function,

$$f_d(x, t) = \frac{1}{\sqrt{2\pi\sigma x\sqrt{t}}} e^{-\frac{(\ln x - at)^2}{2\sigma^2 t}} \quad (16)$$

where  $F_d(x, t)$  is the distribution of diameters,  $\Phi(t)$  is the Laplace function (the cumulative Gaussian distribution function),  $a$  and  $\sigma$  are the distribution parameters, and  $t$  is time elapsed after the start of the process.

The next step of our research work was testing the developed model. The testing was conducted at several reference sites on the Usturt Plateau (Fig. 7). Sites 1-4 and 6-8 are located on the sloping plain to the south of the Karabaur Ridge; sites 5 and 9 are located in the western part of the Usturt, 70 km to the south-west of the Ridge. With respect to structural geology, sites 1 - 4 and 6 - 8 are located on the Karabaur anticline roof; sites 5 and 9 are located on its legs. In terms of geomorphology, all the sites represent a structural-denudation plain with a gently rolling surface and embedded *takyrs* depressions.

The testing methodology included the following steps:

- isolation of karst depressions,
- identification of their diameters, average diameters, and locations of the centres,
- verification of the conformity of the spatial distribution of the karst depression centres to the Poisson Law,

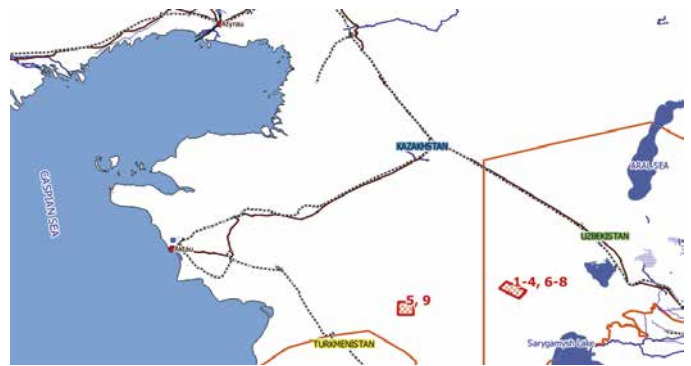


Figure 7: Locations of the sites.  
Figura 7. Ubicación de los sitios de estudio..

- verification of the conformity of the distribution of the diameters and average diameters to the log-normal distribution.

The isolation of the karst depressions at sites 5 and 9 was conducted with the help of satellite images with a resolution of approximately 2 m. For sites 1-3, in addition to the images, we used a map for the karst-suffusion phenomena, compiled earlier (in 1970) by E. A. Agbalyants and V.A. Vazerskaya; the map was compiled from aerial photographs; its spatial resolution is 30-90 cm.

The conformity test of the spatial distribution of the karst depressions to the Poisson law included the following steps:

- random selection of a reference site within an area;
- calculation of the number of the centres of the karst depressions at each site, thus, obtaining a statistical sample; calculation of the average value;
- testing of the sample's conformity to the Poisson distribution using the Pearson criterion ( $\chi^2$ -criterion) and software application Statistica 7.

The conformity test of the diameters and average diameters of the karst depressions to the lognormal law included the following steps:

- identification of the average logarithm of the diameters (average diameter) of the karst depressions and of its root-mean-square value;
- testing of the sample's conformity to the lognormal distribution using the Pearson criterion ( $\chi^2$ -criterion) and software application Statistica 7.

When calculating the Pearson criteria and when necessary, the adjacent intervals were merged in order to provide for at least five sample members in the same interval.

## Results

Analysis of the results of the empirical verification of the spatial distribution of the karst depressions at the sites (Table 1) indicates that, at a 0.99 significance level, the hypothesis on the Poisson distribution of the karst landforms at a randomly selected site corresponds well with the empirical data at three out of four sites.

In addition, multiple analyses were undertaken at site 5. At this site, using a random-number generator, six series of placements was done. Each series included 100 placements. The size of the area

Site #	Average	Sample size	$\chi^2$	$\chi^2_{0,99}$
1	1.384	90	1.677	11.341
2	1.236	110	8.860	9.210
3	1.938	81	3.281	11.341
4	1.862	49	20.903	11.341

**Table 1:** Comparison of the empirical and theoretical Poisson distribution of the karst depressions.

*Tabla 1. Comparación de la distribución empírica y teórica Poisson de las depresiones kársticas.*

was different in each series. The obtained samples were also tested for the Poisson distribution using the Pearson criterion (Table 2). The results indicated that the hypothesis agrees well with the empirical data in all samples. Comparison of the charts of the empirical and theoretical distributions is presented in Figure 8.

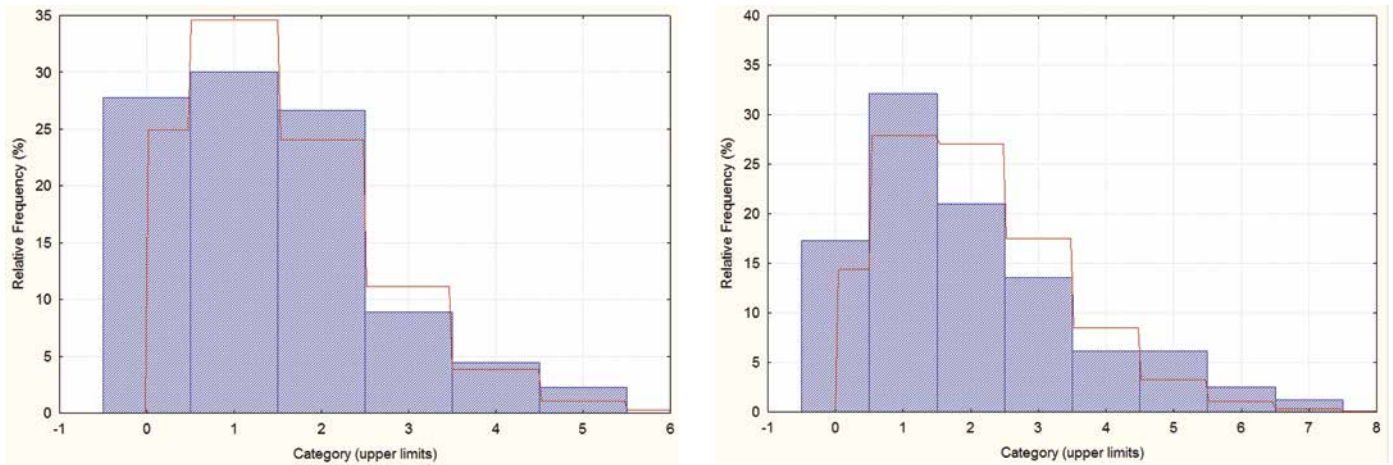
Then, the samples were compared with the theoretical lognormal distribution using the Pearson criterion (Table 2). Analysis of the results (Table 3) indicates that at a 0.99 significance level, the hypothesis on the lognormal distribution corresponds well with the empirical data at three sites and on the lognormal distribution of the average diameters – at all four sites. Comparison of the charts of the empirical and theoretical distributions is presented in Figures 9 and 10. Figure 9 demonstrates correspondence between the diameters of the karst landforms and their theoretical distribution. Figure 10 demonstrates the same correspondence for the average diameters of the karst landforms. The average diameter of a karst landform is arithmetic mean for the diameter and maximum distance between points of the karst landform perpendicular to the diameter. The average

Site #	Series #	Average	Sample size	$\chi^2$	$\chi^2_{0,99}$
5	1	1.390	100	5.60	11.341
	2	0.840	100	6.32	9.210
	3	0.420	100	11.11	6.635
	4	0.840	100	6.25	9.210
	5	0.960	100	5.75	9.210
	6	0.990	100	1.69	9.210

**Table 2:** Comparison of the empirical and theoretical Poisson distribution of the karst depressions for the reference sites of different sizes.

*Tabla 2. Comparación de la distribución empírica y teórica de Poisson de las depresiones kársticas para localidades de referencia de diferentes tamaños.*





**Figure 8:** The empirical and theoretical (Poisson Law) distributions of the number of the centers of the karst depressions at a randomly selected site: Left: site 1. Right: site 3.

**Figura 8.** Las distribuciones empíricas y teóricas (Ley de Poisson) de los diámetros de las depresiones kársticas: distribución del número de los centros de las depresiones kársticas en un sitio seleccionado al azar: izquierda: sitio 1. Derecha: sitio 3.

diameter is less under the influence of random factors than the diameter.

Thus, the research presented in this paper demonstrates that the main patterns of the dimensions and location of the karst landforms also apply to the territories of desert karst. The study proves the validity of both the Poisson law for the spatial location of the karst landforms and the lognormal distribution of the dimensions of the karst landforms. Some deviations may be caused by inhomogeneities of the sites, for example, due to fault displacements.

The two samples showed deviations from theoretical laws. Failure of the Poisson law at site 4 can be explained by the site heterogeneity, which could result from the following factors:

- fracturing of carbonate rocks,
- small unnoticed rock outcrop patches with other dissolubility.

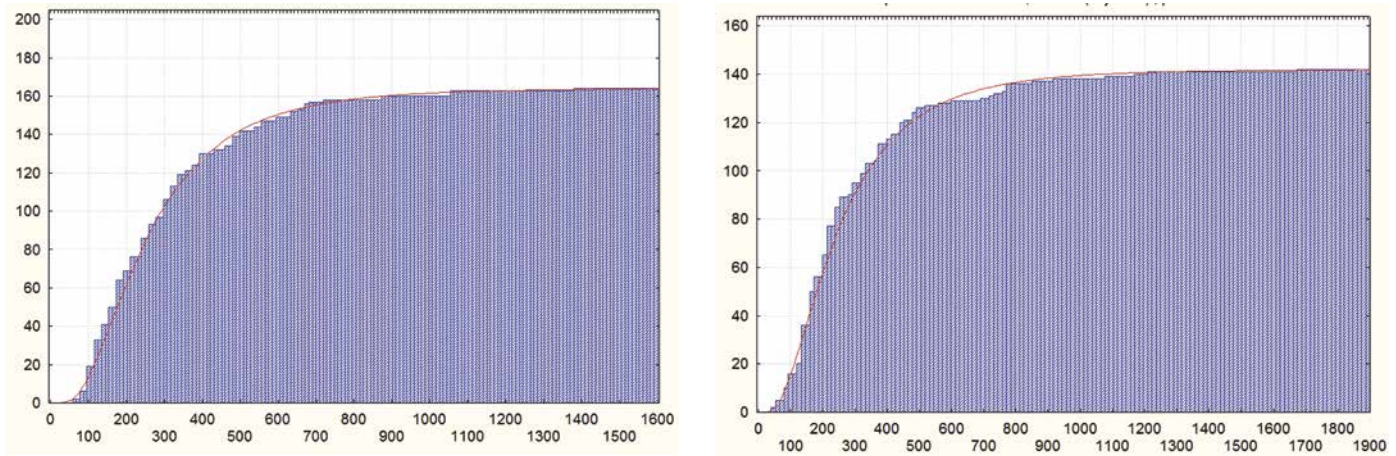
In fact, in some places, near the axis of the Karabaur Ridge for example, faults are significantly expressed.

The same explanation can be used for the deviation from the normal law at site 6. The diameter is generally sensitive to random factors. It should be stressed that although diameter distribution does not correspond to the theoretical distribution, the average diameter distribution obeys the lognormal law.

These results may be used in assessments of karst hazard for linear and areal structures, as well as for small-dimensional structures (Victorov. 2006).

Site	Parameter	$\mu$	$\sigma$	N	$\chi^2$	$\chi^2$
6	diameter	4.519	0.729	149	38.895	18.475
7	diameter	5.153	0.646	165	12.277	21.666
8	diameter	5.092	0.719	142	13.052	15.086
9	diameter	4.632	0.643	52	0.380	9.210
6	average diameter	4.342	0.729	149	18.955	21.666
7	average diameter	4.920	0.578	165	12.429	24.725
8	average diameter	4.905	0.691	142	14.331	21.666
9	average diameter	4.150	0.582	52	1.340	6.535

**Table 3:** Comparison of the empirical and theoretical (lognormal) distributions of the diameters and average diameters of the karst depressions.  
**Tabla 3.** Comparación empírica y teórica (lognormal) de los diámetros y de la media de diámetros de las depresiones kársticas.



**Figure 9:** The empirical and theoretical (lognormal) distributions of the diameters of the karst depressions: Left: site 7; Right: Site 8.  
**Figura 9.** Las distribuciones empíricas y teóricas (lognormal) de los diámetros de las depresiones kársticas: Izquierda: sitio 7. Derecha: sitio 8.

### Conclusions

This research has allowed us to reach the following conclusions:

1. The karst depressions of the Usturt Plateau, which are typical forms of desert karst, are governed by the Poisson distribution of their centres and by the lognormal distribution of their dimensions, similar to the karst depressions in a humid climate.
2. The conditions of the development of karst at a specific site influence only the values of the distribution parameters, whilst the type of the distribution of the centres and dimensions of the karst depressions remain constant.

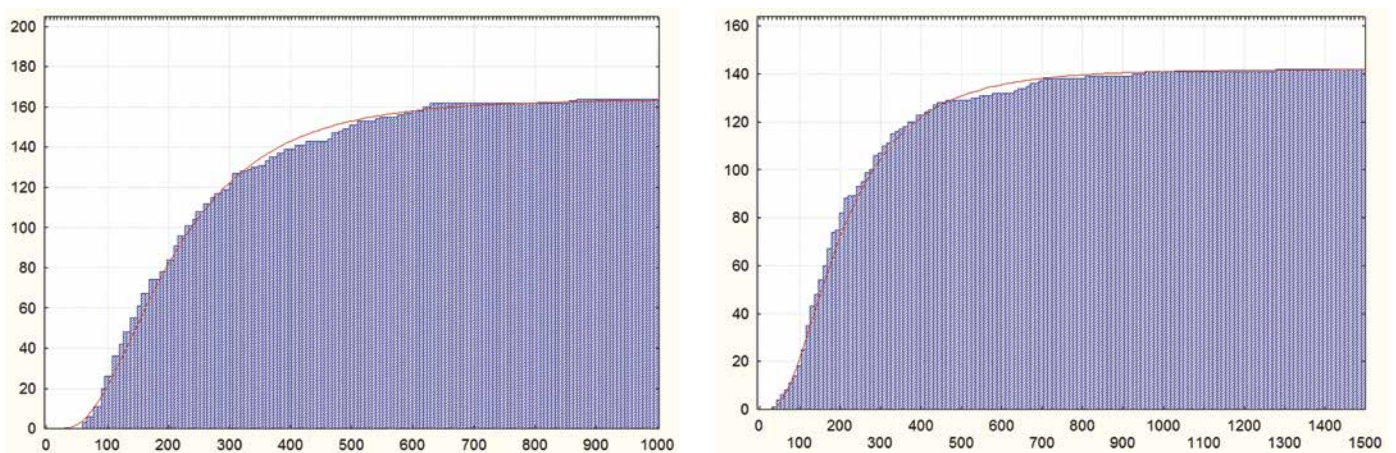
3. In desert karst conditions, the proposed model may be used for the assessment of the probability of damage to linear structures.

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**Figure 10:** The empirical and theoretical (lognormal) distributions of the average diameters of the karst depressions: Left: site 7. Right: site 8.  
**Figura 10.** Las distribuciones empíricas y teóricas (lognormal) de los diámetros medios de las depresiones kársticas: Izquierda: sitio 7. Derecha: sitio 8.

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